Instrument :	Projet de recherche collaborative (PRC)
Champ disciplinaire :	Mathématiques, Sciences du numérique
CES :	Mathématiques, informatique théorique, automatique et traitement du signal
Acronyme :	CLap–CLap
Titre :	La correspondance de Langlands p-adique : une approche constructive et algorithmique

Summary

The p-adic Langlands correspondence has become nowadays one of the deepest and the most stimulating research programs in number theory. It was initiated in France in the early 2000's by Breuil and aims at understanding the relationships between the p-adic representations of p-adic absolute Galois groups on the one hand and the p-adic representations of p-adic reductive groups on the other hand. Beyond the case of $\mathrm{GL}_2(\mathbb{Q}_p)$ which is now well established, the p-adic Langlands correspondence remains quite obscure and mysterious new phenomena enter the scene; for instance, on the $\mathrm{GL}_n(F)$-side one encounters a vast zoology of representations which seems extremely difficult to organize.

The CLap–CLap ANR project aims at accelerating the expansion of the p-adic Langlands program beyond the well-established case of $\mathrm{GL}_2(\mathbb{Q}_p)$. Its main originality consists in its very constructive approach mostly based on algorithmics and calculations with computers at all stages of the research process. We shall pursue three different objectives closely related to our general aim:

(1) draw a conjectural picture of the (still hypothetical) p-adic Langlands correspondence in the case of GL_n,

(2) compute many deformation spaces of Galois representations and make the bridge with deformation spaces of representations of reductive groups,

(3) design new algorithms for computations with Hilbert and Siegel modular forms and their associated Galois representations.

This project will also be the opportunity to contribute to the development of the mathematical software SageMath and to the expansion of computational methodologies.

Members of the project

The head of each node appears in italic.

The percentage indicated in parenthesis corresponds to the percentage of the *research time* dedicated to the project.

Rennes node: *Xavier Caruso* (75%), Agnès David (60%), Reynald Lercier (50%), Élisa Lorenzo-García (33%), David Lubicz (50%), Christophe Ritzenthaler (40%), Matthieu Romagny (75%), Tobias Schmidt (50%)

Paris node: *Ariane Mézard* (60%), Christophe Breuil (50%), Christophe Cornut (25%), Julien Hauseux (25%), Francesco Lemma (50%), Benjamin Schraen (50%)

Lyon node: *Sandra Rozensztajn* (60%), Laurent Berger (40%), Jean-Marc Couveignes (40%), Gabriel Dospinescu (25%), Stefano Morra (75%), Dajano Tossici (33%)

1 Context, positioning, and objective of the full proposal

Roughly speaking, arithmetic geometry aims at solving Diophantine equations by geometric methods. One of its most prominent achievements is certainly the Langlands program, which makes an unexpected connection between representations of the absolute Galois group of $\mathbb{Q}$ and certain adelic representations of reductive algebraic groups. This is significant since many problems in number theory and arithmetic geometry can be reduced to questions concerning Galois groups; the Langlands correspondence then provides new tools of analytic nature for attacking them. On the other hand, for a very long time, an ubiquitous method in number theory was to study problems, objects or equations (*e.g.* Diophantine equations) modulo various integers n or, equivalently, in different fields of p-adic numbers $\mathbb{Q}_p$ where p denotes a prime number.

Let us now fix a prime number p. In the early 2000's, Breuil suggested the existence of a purely p-adic version of the classical local Langlands correspondence. Fifteen years later, the p-adic Langlands correspondence has become a major topic in number theory. This project fits into this dynamic and aims at accelerating the expansion of the p-adic Langlands program. Its main originality consists in its very constructive approach mostly based on algorithmics and calculations with computers at all stages of the research process.

Breuil's initial motivation for looking for a purely local p-adic Langlands correspondence was to understand if p-adic Hodge theory on the Galois side has a counterpart on the GL_n side (it turns out that p-adic Hodge theory is essentially missing in the classical local Langlands program), and if so, then to explain how. In all what follows, fix a finite extension F of $\mathbb{Q}_p$. More precisely, one essential aim of the p-adic Langlands program is to understand the representations of $\mathrm{GL}_n(F)$ in the p-adic or mod p étale cohomology of a tower of Shimura varieties, and to relate them to n-dimensional p-adic or mod p representations of $\mathrm{Gal}(\bar{F}/F)$. It is currently completely understood only in the case of 2-dimensional representations of $\mathrm{Gal}(\bar{\mathbb{Q}}_p/\mathbb{Q}_p)$, that is for the group $\mathrm{GL}_2(\mathbb{Q}_p)$, but even this limited case has had strong applications to modularity theorems: for instance the proofs of the Fontaine–Mazur conjecture for 2-dimensional p-adic geometric representations of $\mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$ rely on the p-adic Langlands program for $\mathrm{GL}_2(\mathbb{Q}_p)$ [78, 56]. If $\mathbb{Q}_p$ is replaced by F, or the dimension 2 by the dimension n, the p-adic Langlands program is essentially open, despite many partial results, in particular recent progress on the locally analytic aspects of the p-adic representations of $\mathrm{GL}_n(F)$.

The objectives of this project consist of three intricated parts: (1) continue to develop the aforementioned locally analytic aspects, (2) study deformation spaces on both sides and try to find in there evidences or hints for the existence and the construction of the p-adic Langlands correspondence and (3) study the cohomology of Hilbert and Siegel modular varieties in which the p-adic Langlands correspondence is expected to be realized.

The main originality of our project builds on our plan to constantly rely on algorithms and computers. We are convinced that the time is ripe to develop this approach. Indeed, on the one hand, several recent results in p-adic Langlands program have highlighted new unexpected phenomena, which are widely *not* understood today. The need to study a large range of new test examples is therefore manifest. On the other hand, we believe that algorithms in number theory are nowadays sufficiently mature and can definitely help us in computing and analyzing large families of required examples. In this sense such algorithms can be decisive elements in future research.

1.1 State of the art

1.1.1 Number theory and algorithmics: a long history

Many interesting questions in number theory are liable to experimental study. This is the case in algebraic number theory: properties of Galois modules [91], search for number field extensions of small discriminant [36], small regulator [116], statistical study of class groups in extensions of a certain kind [38], *etc*. The explicit side of analytic number theory (study of the zeroes of the Riemann zeta function [100]), of diophantine geometry (rational points on surfaces [88], BSD conjecture [118]) and of the geometry of numbers (classification of lattices [92], estimation of the Hermite constant) lead to sometimes delicate computational questions. Effectively solving these questions makes it possible to test and refine conjectures and estimates of constants involved in many number-theoretic inequalities.

The first answer to all these questions consists in constructing tables and making them available to colleagues and users. Number theory software packages were erstwhile designed so as to establish these tables. Since the opening of computers to the masses, the distribution of efficient and uncomplicated programs has superseded the publication of tables. Progress in theoretical computer science, and especially in complexity theory [102, 21], has turned algorithmic number theory into a subject of its own instead of a mere tool. For instance, determining which elementary arithmetic problems can be solved in polynomial or quasi-linear time in a deterministic or probabilistic way is a question which is often difficult and sometimes profound [3].

Nowadays, many advanced functionalities have been implemented: for the study of number fields (computation of the ring of integers, of the class group and of the unit group, evaluation of the Dedekind zeta function), class field theory (computation of ray class fields), elliptic curves (minimal model, heights, L series, modular parametrisation, analytic rank, Heegner points...), lattices, usual transcendental and p-adic functions, and so on. More recent work has focused on the algorithmic aspects of algebraic curves (jacobian varieties, Theta functions, modular equations) and their applications [39], on elementary algorithmic and on cryptography, as well as the computation of Galois representations.

The first step towards a computational Langlands correspondence has been the development and implementation of efficients algorithms for relative number fields and class field theory [35]. Ray class fields are constructed using algebraic methods (Kummer theory) or analytic methods (Stark units).

The second step is concerned with the algorithmic aspects of modular symbols, modular curves, and modular forms. This theory was developed by Birch, Manin, Shokurov, Cremona, Merel among others [49]. The Taniyama–Weil conjecture was thus made effective a long time before it was proven. This results in the existence of algorithms to compute the p-th coefficient of a given modular form in time polynomial in p.

More recent algorithms compute coefficients of modular forms in time polynomial in $\log p$, using efficient computation of the associated modular representations to the given modular form. The algorithm imagined by Couveignes and Edixhoven in [54] to compute Galois representations attached to classical modular forms has been recently improved by Nicolas Mascot [93, 94]. He implemented it in SAGEMATH and used it to compute explicitly the number fields cut out by the mod ℓ representations attached to the newforms of level 1 and weight k for ℓ up to 31, and he was also able to use it to compute the coefficients a_p of these forms mod ℓ for a few primes p of a thousand digits each. This is significant progress towards making efficient the Galois representation side of the Langlands program. Besides, pieces of code and experience can be re-used for the CLap–CLap project: explicit computations with modular jacobians, explicit computations in the homology and cohomology of modular curves, interplay between the algebraic model of the jacobian and the analytic one, reconstruction of algebraic numbers from approximations, stability of linear algebra algorithms over non-exact domains, output certification machinery, to name a few.

Regarding the p-adic side, let us mention that Caruso, Roe and Vaccon recently design a new theory for tracking precision optimally while computing with p-adic objects [32]. This promising theory has been already successfully used for analyzing the loss of precision in many standard primitives (*e.g.* Gauss elimination, Euclidean algorithm) and some more complex algorithms (*e.g.* Gröbner bases, p-adic differential equations). No doubt that it will play a pivotal role in the CLap–CLap project as well.

1.1.2 p-adic Langlands correspondence

In 1998, Harris–Taylor [64] and independently Henniart [72], proved the local Langlands correspondence for $\mathrm{GL}_n(F)$. Recall that it is (essentially) a correspondence between isomorphism classes of smooth (*i.e.* locally constant) irreducible representations of $\mathrm{GL}_n(F)$ over $\mathbb{C}$ and isomorphism classes of certain n-dimensional smooth representations of the Weil–Deligne group of F over $\mathbb{C}$. A few years later, Vignéras proved in [121] that, if one replaces the Weil–Deligne group of F by $\mathrm{Gal}(\overline{F}/F)$, then for any prime number $\ell \neq p$ one can reformulate this correspondence in terms of ℓ-adic instead of smooth representations, *i.e.* there is a bijection between isomorphism classes of continuous topologically irreducible unitary representations of $\mathrm{GL}_n(F)$ on an ℓ-adic Banach space and certain n-dimensional continuous ℓ-adic representations of $\mathrm{Gal}(\bar{F}/F)$ (both over a finite extension of $\mathbb{Q}_\ell$). Here unitary means that the Banach topology is given by a norm which is invariant under $\mathrm{GL}_n(F)$. The main point in the proof is that every continuous (admissible) representation of $\mathrm{GL}_n(F)$ on an ℓ-adic Banach space has a dense subspace of smooth vectors, *i.e.* fixed by a sufficiently small open compact subgroup of $\mathrm{GL}_n(F)$. This ultimately implies that one can essentially derive this ℓ-adic correspondence directly by "completion" from the classical smooth one.

Replacing $\ell \neq p$ by $\ell = p$ is basically the aim of the p-adic Langlands program. However one can at once point out big differences with the $\ell \neq p$ case. On the $\mathrm{GL}_n(F)$ side, continuous (admissible) unitary representations of $\mathrm{GL}_n(F)$ on p-adic Banach spaces in general do not possess a dense subspace of smooth vectors. They rather contain a dense subspace of *locally $\mathbb{Q}_p$-analytic vectors* (which are those vectors for which the orbit map is a locally $\mathbb{Q}_p$-analytic function on $\mathrm{GL}_n(F)$) thanks to a hard theorem of Schneider–Teitelbaum [112]. Therefore, one cannot only rely on the smooth correspondence as for $\ell \neq p$, and any sort of relation with n-dimensional continuous p-adic representations of $\mathrm{Gal}(\bar{F}/F)$ is necessarily a new phenomenon. On the $\mathrm{Gal}(\bar{F}/F)$ side, there is a rich theory of finite dimensional p-adic representations (much richer than ℓ-adic representations when $\ell \neq p$), mainly by the work of Fontaine, after Tate and others. So both theories on the $\mathrm{GL}_n(F)$ and $\mathrm{Gal}(\bar{F}/F)$ sides are richer, and somehow one hopes that these two phenomena are related. In particular one hopes to recover (in a way or another) Fontaine's theory on the $\mathrm{GL}_n(F)$ side. Finally, another important aspect is that unitary Banach space representations of $\mathrm{GL}_n(F)$ by definition possess unit balls which are preserved by the $\mathrm{GL}_n(F)$ action, and that one can reduce modulo p (or modulo an uniformizer of the coefficient ring), thus producing (hopefully more amenable) smooth representations of $\mathrm{GL}_n(F)$ over finite extensions of $\mathbb{F}_p$.

To sum up, one has the following picture on the $\mathrm{GL}_n(F)$ side:

$$\begin{tikzcd}
 & \boxed{\text{Unitary Banach representations}} \arrow[dl, "\text{loc. analytic vectors}"'] \arrow[dr, "\text{reduction modulo } p"] & \\
\boxed{\text{Locally analytic representations}} & & \boxed{\text{Smooth modular representations}}
\end{tikzcd}$$

Since 2000, mainly by the work of Colmez, following the initial work of Breuil and others, a full local p-adic Langlands correspondence between (some) admissible unitary Banach space representations of $\mathrm{GL}_2(\mathbb{Q}_p)$ and 2-dimensional p-adic representations of $\mathrm{Gal}(\bar{\mathbb{Q}}_p/\mathbb{Q}_p)$ has been established, and the above local picture is essentially complete, that is, one can also describe the locally analytic and mod p representations of $\mathrm{GL}_2(\mathbb{Q}_p)$ [9, 10, 40, 6, 5, 79, 45, 42, 105, 53, 41]. This local p-adic Langlands correspondence satisfies moreover two fundamental compatibilities without which it would have a limited interest: it is compatible with the classical local Langlands correspondence and it is compatible with the global theory.

Compatibility with the classical local Langlands correspondence. When the 2-dimensional p-adic representation ρ_p of $\mathrm{Gal}(\bar{\mathbb{Q}}_p/\mathbb{Q}_p)$ is potentially semi-stable (which is basically the most important case!), one can then associate to it using Fontaine's theory a smooth 2-dimensional representation of the Weil–Deligne group of $\mathbb{Q}_p$, which therefore corresponds to a smooth representation π_p of $\mathrm{GL}_2(\mathbb{Q}_p)$ by the classical local correspondence (suitably normalised). When moreover the two Hodge-Tate weights $a \leq b$ of ρ_p are distinct, one can consider the following locally algebraic representation of $\mathrm{GL}_2(\mathbb{Q}_p)$ (over a finite extension E of $\mathbb{Q}_p$) :

$$\det{}^a \otimes_E \mathrm{Sym}^{b-a-1}(E^2) \otimes_E \pi_p. \tag{1}$$

Then one key property of the unitary Banach space representation of $\mathrm{GL}_2(\mathbb{Q}_p)$ corresponding to ρ_p is that its subspace of locally algebraic vectors is nonzero and isomorphic to (1) [41, 56].

Compatibility with the global theory. It turns out that arithmetic geometry provides natural p-adic Banach spaces equipped with commuting continuous actions of both $\mathrm{GL}_2(\mathbb{Q}_p)$ and $\mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$, the so-called *completed cohomology spaces* (first considered by Ohta and Emerton):

$$\Big(\varprojlim_m \varinjlim_n H^1_{\text{ét}}\big(Y(Np^n)_{\bar{\mathbb{Q}}}, \mathcal{O}_E/p^m\mathcal{O}_E\big)\Big) \otimes_{\mathcal{O}_E} E$$

where E is a finite extension of $\mathbb{Q}_p$, N is prime to p and $Y(Np^n)_{\bar{\mathbb{Q}}}$ is the usual modular curve over $\bar{\mathbb{Q}}$ of full level $\Gamma(Np^n)$. Let $\rho : \mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q}) \to \mathrm{GL}_2(E)$ be a continuous, odd, absolutely irreducible, almost everywhere unramified representation and assume that the ρ-isotypic subspace:

$$\mathrm{Hom}_{\mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})}\bigg(\rho, \Big(\varprojlim_m \varinjlim_n H^1_{\text{ét}}\big(Y(Np^n)_{\bar{\mathbb{Q}}}, \mathcal{O}_E/p^m\mathcal{O}_E\big)\Big) \otimes_{\mathcal{O}_E} E\bigg) \tag{2}$$

is nonzero (*e.g.* this holds if $\rho = \rho_f$ for some cuspidal eigenform f of weight ≥ 2 and prime-to-p level N). Then one key property of the local p-adic correspondence is that (2) is isomorphic as a $\mathrm{GL}_2(\mathbb{Q}_p)$-representation to a finite direct sum of the p-adic unitary Banach space associated to $\rho_p = \rho|_{\mathrm{Gal}(\bar{\mathbb{Q}}_p/\mathbb{Q}_p)}$ [11, 55, 56].

We now switch to the state of the art for other groups than $\mathrm{GL}_2(\mathbb{Q}_p)$, more precisely to the part which is relevant for this project. In fact, none of the previous results for $\mathrm{GL}_2(\mathbb{Q}_p)$ have yet been fully extended to other groups, for instance $\mathrm{GL}_3(\mathbb{Q}_p)$, or $\mathrm{GL}_2(F)$ with $F \neq \mathbb{Q}_p$ (not to mention $\mathrm{GSp}_4(\mathbb{Q}_p)$, etc.), but there are many partial nontrivial results on the mod p and locally analytic aspects. We briefly describe the state of the art for these two aspects, which involve quite different techniques but are in the same time complementary.

The classification of admissible irreducible smooth representations of $\mathrm{GL}_n(F)$ over a finite extension of $\mathbb{F}_p$ is a quite difficult question as soon as $n > 2$ or $F \neq \mathbb{Q}_p$. The main result of [73] reduces this problem to the classification of the *supersingular* representations, *i.e.* those admissible irreducible representations which are not subquotients of strict parabolic inductions. However, it is shown in [20] that, at least for $n = 2$, there are far too many smooth admissible irreducible representations of $\mathrm{GL}_2(F)$ over a finite extension of $\mathbb{F}_p$ as soon as $F \neq \mathbb{Q}_p$, killing the hope of a local correspondence with 2-dimensional irreducible representations of $\mathrm{Gal}(\bar{F}/F)$ analogous to the $F = \mathbb{Q}_p$ case. Moreover, the main result of [115] worsen the situation showing that, at least when $[F : \mathbb{Q}_p] = 2$, none of these admissible irreducible supersingular representations of $\mathrm{GL}_2(F)$ are of finite presentation, which means that one needs an infinite number of equations to define them. Such a situation in representation theory seems unprecedented, and so far it is not clear how to deal with it. Should one work in the context of derived categories? Should one only consider some specific representations of $\mathrm{GL}_n(F)$, *e.g.* those which appear in a global context? Indeed, the Galois isotypic subspaces in the mod p cohomology (for instance the analogue of (2) for $n = 2$ and quaternionic Shimura curves over a totally real field) do provide some interesting specific smooth admissible representations of $\mathrm{GL}_n(F)$, and their $\mathrm{GL}_n(\mathcal{O}_F)$-socle has been the subject of an extensive study over the recent years [59]. Moreover, these specific admissible representations of $\mathrm{GL}_n(F)$ do seem to determine the local representation of $\mathrm{Gal}(\bar{F}/F)$ [14, 113, 74, 103]. One crucial open question however is that no-one knows if they are of finite length as soon as $\mathrm{GL}_n(F) \neq \mathrm{GL}_2(\mathbb{Q}_p)$.

Since one does not understand correctly mod p representations of $\mathrm{GL}_n(F)$, even those carried by the mod p cohomology, one understands even less those p-adic Banach spaces representations of $\mathrm{GL}_n(F)$ carried by the completed p-adic cohomology. However, there has recently been progress on their locally analytic vectors. One reason is that locally analytic representations of $\mathrm{GL}_n(F)$ tend to have far more constituents than

p-adic Banach spaces or mod p representations (*e.g.* they are related to Verma modules, which can have many constituents with complicated multiplicities). Though one certainly doesn't understand *all* the locally analytic vectors of the completed cohomology, one understands enough to discover new and interesting phenomena. For instance, the (finite slope part of the) locally analytic socle of the Galois- (or Hecke-) isotypic subspaces in the crystalline case has been recently determined in [17], by using and studying the geometry of various rigid analytic varieties called Eigenvarieties, which can be roughly seen as moduli spaces for p-adic overconvergent automorphic forms of finite slope or for trianguline n-dimensional representations of $\mathrm{Gal}(\bar{F}/F)$. The geometry of these Eigenvarieties also led to almost optimal results on the classicality of overconvergent automorphic forms whose Galois representation is crystalline at p ([15], [16], [17]). In quite another direction, there has been results providing a localization of locally analytic representations quite analogous to the classical localization of Lie algebra representations as $\mathcal{D}$-modules on flag varieties [106, 76]. In the case $n = 2$, this localization process already has had one application: the proof that those locally analytic representations of $\mathrm{GL}_2(F)$ coming from the (dual of) the rigid analytic de Rham complex of the first Drinfeld covering are admissible in the sense of [112].

1.2 Our objectives

The main goal of our project consists in finding new evidences towards the existence of a p-adic Langlands correspondence beyond the nowadays well-established case of $\mathrm{GL}_2(\mathbb{Q}_p)$. In order to carry out this project, we plan to take inspiration from the very first works of Breuil when he tried to draw the picture of what should be the p-adic Langlands correspondence in the case of $\mathrm{GL}_2(\mathbb{Q}_p)$. Roughly speaking, one may say that his work was a subtle association of three different approaches.

The first approach has consisted in studying separately 2-dimensional p-adic representations of Galois groups and (torsion, locally analytic, Banach) p-adic representations of $\mathrm{GL}_2(\mathbb{Q}_p)$ and, as a second step, in trying to match them. As the best example, this strategy has worked perfectly well in the case of p-torsion representations, yielding the first evidences in favour of the existence of a p-adic Langlands correspondence. Indeed it turns out that both sides are indexed by the same parameters and therefore can be mapped bijectively [9]. The second approach was to study deformation spaces[1]. The idea behind this is rather simple: if Galois representations have to correspond to representations of $\mathrm{GL}_2(\mathbb{Q}_p)$, the deformation spaces on both sides should match. Breuil and Mézard followed this idea and came up with the famous Breuil–Mézard conjecture which states an unexpected relation between numerical invariants attached to Galois deformation spaces and other numerical invariants coming from the theory of representations of $\mathrm{GL}_2(\mathbb{Q}_p)$ [19]. The Breuil–Mézard conjecture nowadays appears as one of the most important corner stones in the p-adic Langlands correspondence. Finally, it is widely expected that the Langlands correspondence should be realized in the cohomology of Shimura varieties which carries at the same time a Galois action and an action of a reductive group. To look for a p-adic correspondence for $\mathrm{GL}_2(\mathbb{Q}_p)$ in the completed cohomology of modular curves forms the third approach [11]. After Breuil's seminal work, this approach was developed by Emerton who managed to derive from it spectacular results and applications [55, 56].

As mentioned before, we plan to follow a similar strategy in the setting of this project, *i.e.* for a reductive group which is *not* $\mathrm{GL}_2(\mathbb{Q}_p)$. Precisely, we plan to pursue the three following objectives :

- **Objective 1:** *Locally analytic aspects for* $\mathrm{GL}_n(F)$

  This objective aims at understanding better the locally analytic representations of $\mathrm{GL}_n(F)$ where F is a finite extension of $\mathbb{Q}_p$ and at trying to match (some of) them with n-dimensional p-adic representations of $\mathrm{Gal}(\bar{F}/F)$. (Note that the Galois side is actually rather well understood *via* p-adic Hodge theory.)

[1] In fact this approach really took shape later, with the work of Colmez, Emerton and Kisin [40, 55, 79].

- **Objective 2:** *Deformation spaces*

  This objective aims at studying several deformation spaces on both sides and at trying to compare them. More results are expected on the Galois side for which the technology is much better established. We nevertheless plan to work on the GL_n-side as well and expect to obtain precise descriptions in the simplest cases (*e.g.* deformations of parabolic inductions).

- **Objective 3:** *Hilbert and Siegel modular forms*

  This objective aims at understanding the cohomology of Hilbert and Siegel modular varieties (which are among the simplest Shimura varieties beyond modular curves). As a matter of priority, we plan to focus on Galois representations associated to Hilbert and Siegel modular newforms since there are simplier objects which can be handled more easily.

Each of the above objectives focuses on a limited set of pieces of the giant puzzle of the p-adic Langlands program and is supposed to shed partial light on the correspondence. From this angle it seems quite clear that these different objectives will have rather deep interconnections.

In order to carry out our project, we plan to make an intensive use of computers and algorithmical methods. Of course, we plan to use computers as a daily "collaborator" in order to perform routine calculations and computing small examples. This will help us a lot in building a good intuition and testing conjectures. However we also plan to go further in this direction and write optimized SAGEMATH packages for solving specific interesting problems (*e.g.* the computation of Galois representations associated to Hilbert modular forms) and share them with the community. These packages will also allow us to compute a huge amount of new examples and feed usual databases (*e.g.* LMFDB, `http://www.lmfdb.org/`).

We refer to §2.1 for much more details.

1.3 National and international positioning of the project

The local p-adic Langlands correspondence was born in France 10 years ago and, since then, it has been a peak of excellence of French mathematical research. In few years it has become a major subject of research all over the world with a positive financial politic (especially in the US) in order to invite and keep the raising stars in the field. Similarly France has a strong tradition in symbolic computation, highly recognized all over the world (see for instance the list of accepted papers at the ISSAC conference). The goal of the CLap–CLap project is to give the opportunity to French scientists to maintain their position as pioneers in this flourishing field of research.

The CLap–CLap team gathers French experts with a worldwide network of ongoing collaborations: X. Caruso with the Massachusetts Institute of Technology (USA) and the University of Waterloo (Canada), L. Berger with the University of Münster (Germany) and the University of Tokyo (Japan), C. Breuil and S. Morra with the University of Toronto (Canada) C. Breuil and B. Schraen with the University of Münster (Germany), F. Lemma with Osaka University (Japan), S. Morra with the University of Arizona and Northwestern (USA) and KIAS (South Corea), ... With the Clap-CLap project we aim at keeping and further expanding our position as worldwide leaders.

The explicit Langlands correspondence is an important emerging subject. As a prominent example, note that a special trimester intitled "Computational Aspects of the Langlands Program" was organized at Brown University two years ago. Beyond that, every year, almost 100 conferences on arithmetic geometry take place. About 25% of them are directly related to p-adic Langlands program, witnessing how groundbreaking this subject is to the world mathematical community.

The explicit p-adic Langlands correspondence is a very new aspect connecting two different communities which have no chance to meet in the 75 conferences scheduled in arithmetic geoemtry in 2016. Much more than a p-adic numerical version, it is a promising and already successful new subject of research.

2 Scientific and technical program, project organisation

2.1 Detailed research project

We now give more details on our research project according to the three main objectives defined in §1.2. For each objective we isolate about half a dozen precise tasks. Moreover we evaluate and indicate the difficulty of each task using the following score:

★☆☆: one can expect a complete solution for this task (main tools are already available in the literature);

★★☆: one can expect huge progress on this task but some corner cases might remain out of reach;

★★★: one can expect substantial progress on this task but probably not a complete solution.

Dependances between the different tasks appear on the Gantt diagram, page 15.

Before detailing each objective, we state a transversal task which is pivotal in the project since p-adic rigid geometry is nowadays ubiquitous in almost all topics of the p-adic Langlands correspondence.

Task T 0.1 — DIFFICULTY: ★⯪☆

Design and implement optimized algorithms for manipulating p-adic rigid varieties.

We plan to attack Task T 0.1 by combining two ingredients: first, the recent works of Vaccon on p-adic Gröbner bases [120] and second, the classical theory of standard bases which deals with formal series over the reals.

Since p-adic geometry has a huge interest beyond the p-adic Langlands correspondence, we would like to spend some time to optimize and package our implementation. We will then propose it for inclusion in SageMath.

2.1.1 Objective 1: Locally analytic aspects for $\mathrm{GL}_n(F)$

Thanks to the recent results mentioned at the end of §1.1.2, many questions and problems arise of various difficulties. We mention several of them in what follows. If $\overline{r} : \mathrm{Gal}(\overline{F}/F) \to \mathrm{GL}_n(k_E)$ is a continuous local Galois representation, we recall that there exists a rigid analytic variety $X_{\mathrm{tri}}(\overline{r})$ defined in [70] and [15, §2.2] which is (essentially) a moduli space for (refined) trianguline deformations of $\overline{r}$. It comes with a rigid analytic morphism towards the "extended" weight space, that is, the rigid analytic variety parametrising the locally analytic characters of the diagonal torus of $\mathrm{GL}_n(F)$.

Task T 1.1 — DIFFICULTY: ★☆☆

Generalize the adjunction formula of [12] for the locally analytic Jacquet–Emerton functor first from finite length locally algebraic representations (of the Levi subgroup) to more general finite length locally analytic representations (using results of Orlik–Strauch), and then in families.

Task T 1.2 — DIFFICULTY: ★★☆

Extend the results on the locally analytic socle and on companion points of [16], [17] from crystalline representations of $\mathrm{Gal}(\overline{F}/F)$ to more general trianguline representations, using (and generalizing) the local model of [17]. Stratify the rigid variety $X_{\mathrm{tri}}(\overline{r})$ in terms of these (generalized) companion points and study the properties of this stratification.

Task T 1.3 — DIFFICULTY: ★★☆

Prove new cases of the locally analytic "Breuil–Mézard–Kisin–Emerton–Gee" conjecture of [17] relating the locally analytic socle to the geometry of fibers over the weight space of the rigid variety $X_{\mathrm{tri}}(\overline{r})$. This conjecture is known in the crystalline case [17], but the more general trianguline situation is open. This task should be closely related to Task T 1.2.

Task T 1.4 — DIFFICULTY: ★★☆

Using the local model of [17], study explicitly the singularities of the rigid variety $X_{\mathrm{tri}}(\overline{r})$ at classical crystalline critical points. If $n = 2$, it is smooth, and if $n = 3$, one can prove by computing explicit equations that the only possible singularity is a determinantal singularity (in particular it is not Gorenstein). Can one obtain similar results in higher dimension, possibly with the help of a computer?

Task T 1.5 — DIFFICULTY: ★☆☆

Determine, at least conjecturally, the whole finite slope part of the completed cohomology in the crystalline case, where, by definition, the finite slope part is the maximal subrepresentation such that all its irreducible constituents are subquotients of locally analytic principal series. Recall that its socle is known (in the crystalline case). Note that, for $n \geq 3$, the finite slope part might involve Galois parameters because constituents of its socle might reappear "in the middle" (as what happens with $K(1)$-invariants of mod p cohomology in recent results of Le-Le Hung-Morra).

Task T 1.6 — DIFFICULTY: ★★☆

Extend the conjectures of [13] on the finite slope part in the semi-stable noncrystalline case. In particular, starting from a local continuous representation $r : \mathrm{Gal}(\bar{\mathbb{Q}}_p/\mathbb{Q}_p) \to \mathrm{GL}_n(E)$ which is semi-stable of Steinberg type (that is, such that $N^{n-1} \neq 0$ on $D_{\mathrm{st}}(r)$, N being the monodromy operator), define a locally analytic representation of $\mathrm{GL}_n(\mathbb{Q}_p)$ that has a chance both to sit inside the completed cohomology and to completely determine the Hodge filtration on $D_{\mathrm{st}}(r)$.

Task T 1.7 — DIFFICULTY: ★★★

Extend the admissibility results of [106] (for locally analytic representations of $\mathrm{GL}_2(F)$ coming from the isotypic subcomplexes of the de Rham complex of the first Drinfeld covering of the p-adic upper half plane) first from the first covering to any Drinfeld covering, and then from dimension 2 to arbitrary dimension n using the results of [76].

The above list is clearly not exhaustive. For instance one can also study the question of the existence of invariant norms on irreducible locally analytic representations of $\mathrm{GL}_n(F)$, *e.g.* on those appearing in the locally analytic socle (see Task T 1.2 above), which includes the case of locally algebraic representations (see [23, §5]), or the (more interesting and harder) question of the multiplicity of the irreducible constituents in the locally analytic socle (in the crystalline case).

2.1.2 Objective 2: Deformation spaces

The p-adic local Langlands program for $\mathrm{GL}_n(F)$ calls for a correspondence between certain n-dimensional p-adic representations of the absolute Galois group G_F and certain p-adic representations of $\mathrm{GL}_n(F)$. Thus if two representations are related by the correspondence, certainly also their universal deformation rings should be related. The purpose of this Objective is to search for "numerical" evidences of this matching by computing (and comparing) deformation spaces on both side.

We first focus on the Galois side. Let E be a sufficiently large extension of $\mathbb{Q}_p$, $\mathcal{O}_E$ be its rings of integers and k_E be its residue field. Given a representation $\bar{\rho} : G_F \to \mathrm{GL}_d(k_E)$ (assumed absolutely irreducible for simplicity), a characted $\psi : G_F \to \mathcal{O}_E^\times$ and a Weil–Deligne representation $\mathbf{t}$, Kisin constructed a ring $R^\psi(\mathbf{t}, \bar{\rho})$ whose E-valued points are in bijection with the lattices inside semistable representations of determinant ψ, Hodge–Tate weights $\{0, 1\}$ for each embedding, Galois type $\mathbf{t}$ and that reduces to $\bar{\rho}$ modulo the maximal ideal. Kisin's strategy was to construct an auxiliary space $\mathcal{K}^\psi(\mathbf{t}, \bar{\rho})$ which is a formal scheme parametrizing semi-linear algebra objects called Breuil–Kisin modules and then build $\mathrm{Spf} R^\psi(\mathbf{t}, \bar{\rho})$ from it. Kisin's construction yields a morphism $f : \mathcal{K}^\psi(\mathbf{t}, \bar{\rho}) \to \mathrm{Spf} R^\psi(\mathbf{t}, \bar{\rho})$ which can be thought of as a partial resolution of singularities.

Kisin proved moreover that f induces an isomorphism over the generic fibre.

Caruso, David and Mézard revisited these ideas in order to obtain an explicit description of $R^\psi(\mathbf{t}, \bar{\rho})$ when $\bar{\rho}$ has dimension 2 and the Galois type $\mathbf{t}$ is tamely ramified [30, 31]. The first step of their strategy is to study the special fibre of $\mathcal{K}^\psi(\mathbf{t}, \bar{\rho})$, a so-called Kisin variety. In the second step, inspection of the specialization morphism from generic fibre to special fibre allows them to describe a geometric construction of a candidate for $R^\psi(\mathbf{t}, \bar{\rho})[1/p]$.

Task T2.1 — DIFFICULTY: ★⯪☆

Make the construction of [31] more explicit: find the equations of the candidates described in *loc.cit*, exhibit a natural integral subring in it (a candidate for $R^\psi(\mathbf{t}, \bar{\rho})$) and describe its special fibre.

Focusing on the special fibre is relevant because we already know a lot about it thanks to the Breuil–Mézard conjecture —which is proved in this setting [63]. Checking if these properties hold then appears as a good test for our candidates.

We then plan to study other various situations: in higher dimension, with higher Hodge–Tate weights and/or with wildly ramified Galois types. We plan to rely on Fargues' point of view on Kisin modules [58], which elucidates most of the theoretical side of the question (though it involves more complicated modules which are consequently more difficult to implement). We split our research project into two tasks.

Task T2.2 — DIFFICULTY: ★⯪☆

Initiate a more systematic study and explicit description of Kisin varieties, which notably includes the treatment of other reductive groups than GL_n.

Task T2.3 — DIFFICULTY: ★★★

Compute moduli spaces of Breuil–Kisin–Fargues modules, glue them and guess candidates for the (generic fibre of the) corresponding Galois deformation rings. In the simplest examples, this task could be attacked by carrying all computations on computers after Task T0.1 will be completed.

In a work in progress, Le, Le Hung, Levin and Morra constructs a geometrical object, called a "local model", in which Kisin varieties and Galois deformations rings live. This "local model" has one more important feature: it naturally reveals the notion of Serre's weights and then connects the Galois side to the GL_n-side in the spirit of the Breuil–Mézard conjecture (and eventually will provide a proof of it). For us, it can serve as a good test for our candidates:

Task T2.4 — DIFFICULTY: ★★★

For each candidate found in T2.3, embed it in the aforementioned "local model" and compare the Serre's weights obtained this way with those predicted by the Serre-type conjectures.

All the complexity of the resolution $f : \mathcal{K}^\psi(\mathbf{t}, \bar{\rho}) \to \mathrm{Spf}\, R^\psi(\mathbf{t}, \bar{\rho})$ comes from the fact that the finite flat group scheme which gives rise to a Galois representation is not uniquely determined. In order to understand the underlying phenomena, we may want to consider first an analogue in equal characteristic, which is supposed to be simpler. Kisin varieties are then replaced by moduli spaces of integral models of a given Dieudonné module M_F defined over a complete discretely valued field F. The latter naturally lives in the affine Grassmannian and is likely within reach.

Task T2.5 — DIFFICULTY: ★☆☆

Construct the aforementioned moduli space $P(M_F)$ as a closed subspace of the Greenberg space of level n of the affine Grassmannian Gr. For bounded level in the ind-system, compute an explicit description (*e.g.* equations) for this moduli space.

In order to understand the geometry and topology of $P(M_F)$, we promote the point of view of buildings. We embed our space inside the Bruhat-Tits building $\mathcal{B}(\mathrm{GL}_n)$. In the paper [47], a similar space for models of an isocrystal β is studied and described as a suitable neighbourhood of a skeleton identified with the building $\mathcal{B}(J)$ where $J = \mathrm{Aut}(\beta)$. This leads to address the following.

Task T2.6 — DIFFICULTY: ★★☆

Find a similar building-theoretic description of $P(M_F)$ in terms of the group of semi-linear automorphisms of the module M_F. Study the components and the singularities of $P(M_F)$ in light of this description.

On the $\mathrm{GL}_n(F)$-side, we would like to develop further the deformation theory for smooth mod p representations $\bar{\pi}$ of $\mathrm{GL}_n(F)$ or more general p-adic reductive groups. Suppose for simplicity that $\bar{\pi}$ has trivial endomorphisms. In contrast to the Galois side, the structure theory for the deformation ring $R_{\bar{\pi}}$ is still in its infancy and not much is known besides the case of $\mathrm{GL}_2(\mathbb{Q}_p)$. Recent progress is made, relying on results in [65], for the structure of $R_{\bar{\pi}}$ when $\bar{\pi}$ is a parabolically induced representation of $\mathrm{GL}_n(F)$ [68] or a generalized Steinberg representation [69].

Task T2.7 — DIFFICULTY: ★★⯪

Find explicit descriptions (*e.g.* equations) of the deformations ring $R_{\bar{\pi}}$ in the simplest non-trivial cases and match them with those obtained on the Galois side.

2.1.3 Objective 3: Hilbert and Siegel modular forms

The guideline of this objective is to design efficient algorithmical solutions for dealing with cohomology of some Shimura varieties in which the p-adic Langlands correspondence should be realized. In such a general setting, this question seems completely unreachable with the current technology. That is why we shall only focus on particular cases, namely that of Hilbert and Siegel modular varieties. Even under this additional restriction, it turns out that the relevant spaces of cohomology are generally quite large (*e.g.* they have infinite dimension over k_E), so that we cannot hope to embrace it as a whole on a computer. Nevertheless they admit interesting finite dimensional Galois invariant subspaces, which are cut out by Hilbert or Siegel modular eigenforms. Designing algorithms for computing these subspaces now appears as a rather realistic (but still ambitious) research project.

Before coming to Hilbert of Siegel modular forms, we focus on the case of classical modular forms. Following a method devised by Couveignes and Edixhoven in [54], Mascot recently designed an efficient algorithm for computing the Galois representations attached to certain modular newforms. Nevertheless, Mascot's algorithm has two important misfeatures regarding the application we have in mind: (1) it does not work when the level is divisible by p and (2) it only deals with p-torsion coefficients. Our first tasks will be to remove these limitations.

Task T3.1 — DIFFICULTY: ★☆☆

Extend Mascot's algorithm for allowing levels which are divisible by p.

Task T3.2 — DIFFICULTY: ★★☆

Extend Mascot's algorithm to more general torsion coefficients, that are $\mathcal{O}_E/\pi_E^n\mathcal{O}_E$ for $n > 1$.

We expect the Task T3.1 to be straightforward. Indeed the current version of Mascot's algorithm does already allow any level in weight 2 and standard theorems should allow us to reduce the general case to that one. For Task T3.2, we plan to argue by induction on n and use Monge's explicit results in Local Class Field Theory [97] for the heredity step. Indeed, if $\rho_n : \mathrm{Gal}(\bar{\mathbb{Q}}_p/\mathbb{Q}_p) \to \mathrm{GL}_2(\mathcal{O}_E/\pi_E^n\mathcal{O}_E)$ is the Galois representation we want to compute and L_n is the extension cutted out by $\ker \rho_n$, it turns out that L_{2n}/L_n is an abelian group whose Galois group is explicit. By explicit Local Field Theory, one should be able to describe L_{2n} from the knowledge of L_n and encode it in a compact form.

We now move to Hilbert modular forms for which our program is quite similar.

Task T 3.3 — DIFFICULTY: ★★☆

> Let f be a Hilbert newform over a totally real number field F and let ℓ be an auxiliary prime number (which may be equal to p). Adapt Mascot's algorithm to compute the mod ℓ reduction of the ℓ-adic representation
> $$\rho_{f,\ell} : \mathrm{Gal}(\bar{F}/F) \longrightarrow \mathrm{GL}_2(K_f \otimes \mathbb{Q}_\ell) \tag{3}$$
> attached to f, where K_f denotes the number field spanned by the Hecke eigenvalues of f.

Let us now discuss briefly our ideas on Task T3.3. Assuming for simplicity that $\ell \neq p$, we can combine a weight-lowering argument (as in the classical case, *cf.* [14, lemma 2.9]) and the Jacquet–Langlands correspondence in order to prove that residual representations of the form (3) are afforded in the ℓ-torsion of the jacobian of a Shimura curve defined over F, where they can thus be "captured". These curves are quotients of the upper half-plane by cocompact fuchsian groups attached to an order in a quaternion algebra over F. They are thus extremely similar to modular curves. The algorithms developed in Page's PhD thesis [101] can determine fundamental domains for these curves. These are basically the inputs that will allow us to extend Mascot's algorithm to the Hilbert case. Some new difficulties are nevertheless expected. For example the lack of cusps would prevent us from using Khuri–Makdisi's algorithms for computing in the jacobian of the aforementioned curves. One should however be able to tackle this issue by replacing cusps with CM points as in Voight–Willis' works [123].

The case of Siegel modular forms seems much more difficult to handle because Siegel varieties have dimension greater than 1 and their geometry has not been completely elucidated (from the algorithmic point of view) yet. The following task then appears as a unavoidable prerequisite.

Task T 3.4 — DIFFICULTY: ★★☆

> Find an explicit and usable description of Siegel varieties and sheaves of modular forms on them equipped with their Hecke action (at least for small genus).

Similar questions were already addressed by several authors [61, 107, 8, 126] but we plan to revisit it using Mumford theory of theta functions. We are convinced that this new approach has the potentiel to lead to breakthrough results. Indeed theta functions are directly related to Siegel modular forms (they actually *are* Siegel modular forms and they can be used to generate them all [80]) and recent works explicit the Hecke action on them [62, 125]. Furthermore quite efficient algorithms for computing and manipulating theta functions on computers were designed and implemented [57, 89].

Task T 3.5 — DIFFICULTY: ★★★

> Given an auxiliary prime number ℓ, compute the mod ℓ representation attached to a Siegel modular newform.

2.2 Work program and resources requested

Members of the project We recall that we have divided our research project into three different objectives. The table presented in Figure 1 (page 13) shows the people involved in the CLap–CLap ANR project distributed according to their node affiliation and their involvement in the project, according to the objectives we have defined. Moreover, we plan to collaborate occasionally with other colleagues disseminated all around the world: Eugen Hellmann (Münster, Germany), Florian Herzig (Toronto, Canada), Cédric Pépin (Paris, France), Peter Schneider (Münster, Germany), Takeshi Tsuji (Tokyo, Japan), Tristan Vaccon (Limoges, France), *etc.*

Work program. The three objectives on which this project focuses are at the same time interconnected and rather self-contained. For this reason, we plan to work on each objective separately. For each objective, a leader will be named at the beginning of the project. His/her task will be to supervise the research related to

	Rennes node Head: X. Caruso	**Paris node** Head: A. Mézard	**Lyon node** Head: S. Rozensztajn
Objective 1	Xavier Caruso Tobias Schmidt	Christophe Breuil Julien Hauseux Ariane Mézard Benjamin Schraen	Laurent Berger Gabriel Dospinescu Stefano Morra
Objective 2	Xavier Caruso Agnès David Matthieu Romagny Tobias Schmidt	Christophe Cornut Ariane Mézard	Gabriel Dospinescu Stefano Morra Sandra Rozensztajn Dajano Tossici
Objective 3	Xavier Caruso Reynald Lercier David Lubicz Christophe Ritzenthaler	Francesco Lemma	Jean-Marc Couveignes Gabriel Dospinescu Stefano Morra

Figure 1: Members of the CLap–CLap project

this objective and to communicate on it. We think that, for each objective, the research may start from the beginning and last until the end of the project.

We plan to develop a highly performant *inter-objective* communication. To this end, we will organize one *inter-objective* meeting per year which (at least) three members per objective (including the leader) will attend. These meetings will be the place to report on progress and share ideas on the one hand, and a unique chance to bring closer two communities who are not used to work together. In the same direction, we would like to give the opportunity to each member of the CLap–CLap ANR project to visit or invite his/her collaborators.

In order to disseminate our results among the community, we plan moreover to organize at least two international conferences. The first one will be part of the thematic semester "Correspondences" organized by the Henri Lebesgue Center (CHL); it will be held in Rennes in September 2019. In addition, we shall apply for a trimester at IHP in 2022 dedicated to p-adic Langlands correspondence. This trimester will provide us facilities for inviting key people from all over the world and organizing intensive working sessions between us. This trimester will cultimate in June 2022 with the final conference of the CLap–CLap project.

The table on the left of Figure 2 (page 14) summarizes the events we plan to organize.

As already mentioned, we plan to make an intensive use of computers for carrying out our research. However computers may also help us in organizing collaborative work and we plan to develop this point. Precisely, if the project is accepted, we will install a friendly interface — probably based on a mixture of COCALC, CHARLATEX and GIT and/or taking advantage of the solutions provided by the public french network MATHRICE — for helping us in writing papers and developing code. We emphasize that we would like to avoid relying on private companies (as GOOGLE or DROPBOX) as much as possible.

Resources requested. In order to carry out our research project with success, we estimate our total needs (including salaries) to about **2600 k€**. Detailed justifications are provided in the joinded financial document. Among this total amount of money, we request a funding of **232 k€** from the ANR.

This funding will be used for different purposes. First of all, we plan to offer one one-year post-doctoral position during the 48 months of the project, to be released at the University of Rennes 1. The nominee will join our team and will be consider as a full member of the project. Depending on his/her qualifications and personal preferences, he/she will join the group working on objective 2 or 3 (or both). We emphasize that the members of the CLap–CLap ANR project are qualified to masterfully supervise this position.

Date	Place	Event
Oct 2018	Lyon	Inter-objective meeting: starting workshop
Sep 2019	Rennes	Conference at CHL semester
Jun 2020	Bordeaux	Inter-objective meeting: focus on PARI and SAGEMATH
Mar 2021	CIRM (?)	Inter-objective meeting
Spring 2022	Paris	Trimester IHP

	Rennes node	Paris node	Lyon node
Post-doc	50 k€	–	–
Traveling & Visiting	20 k€	15 k€	15 k€
Workshops	–	20 k€	20 k€
Conferences	5 k€	30 k€	–
Equipment	6 k€	7 k€	7 k€
Support free publication	5 k€	–	5 k€
Support colloques	4 k€	3 k€	3 k€
Total	90 k€	75 k€	50 k€
University part	7 k€	6 k€	4 k€
Total TTC	97 k€	81 k€	54 k€
	232 k€		

Figure 2: Planified events (on the left) and budget (on the right) of the project

In addition the funding will be used to organize the aforementioned inter-objective meetings and let the members of the CLap–CLap project attend conferences and invite or visit collaborators. On average, we estimate that each member of the project would use the latter possibility once per year. On the contrary, the two aforementioned international conferences will be mostly supported by specific independant fundings: the Centre Henri Lebesgue will support the mid-term conference in September 2019 and we will request fundings to the IHP to support the final trimester.

The members of the CLap–CLap project all strongly believe that free publication is very important for the exemplary development of academic research. For this reason, we want to be able to support all initiatives in this direction (*e.g.* Mersenne's platform).

The table on the right of Figure 2 summarizes the approximative repartition of the requested fundings.

3 Project impact, strategy for valorisation

Scientific, social and economical context

French research in Mathematics holds a world-leading position. The French scientific system is also particularly well connected at an international level. Young PhD of CLap–CLap ANR members got many post-doctoral opportunities in the best departments of mathematics where arithmetic geometry is advanced: King's College, Imperial College, University of Toronto, University of Warwick... From Chicago to London, going through Bonn, the French school of arithmetic geometry is well known and very frequently invited for long terms visits. It then often have to opportunity to disseminate the ideas of the p-adic Langlands program which is really born under French auspices ten years ago.

Unfortunately, the French lattice is not as well organized as the English, German or American systems. Despite its major role, the French school of mathematics keeps its austere recluse reputation. We don't even have strength to recruit post-doctoral students or long terms visiting researcher, to organize annual meetings with the key speakers, to be sure to have a PhD grant early enough to be competitive with the American Graduate school (which may guarantee the PhD grant before the end of the Master). CLap–CLap ANR members have played a full part to ensure proper dissemination of their fields and results (no local recruit, incentive to independence of the students, wide and diversified publications...) but now need to be structured in order to achieve new scientific advances.

Within a few decades, computers have profoundly changed everyday life of people and mathematiciens in

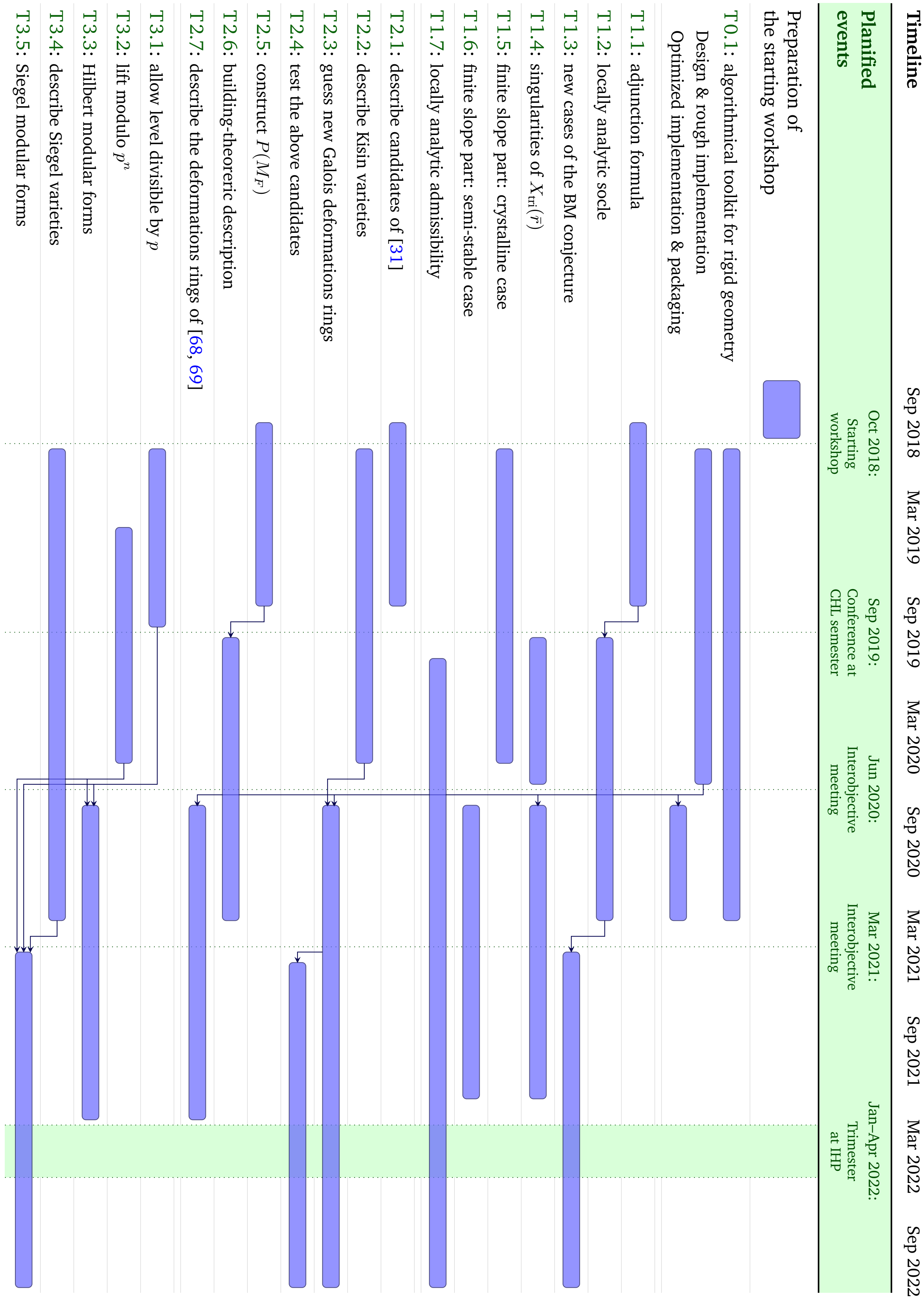

Figure 3: Gantt diagram of the project

particular; recall that the latter used to write papers on a typewriter and did not know electronic mail just 30 years ago. It turns out that computer science is continuing its expansion very quickly. Algorithmics and symbolic computation (which are certainly the two most relevant domains in relation with the CLap–CLap project) have become major topics in computed science and are today actively developed. They nowadays provide softwares which are more and more powerful, easy to use and can manipulate more and more complicated mathematical objects. Beyond that, there were recent spectacular progresses in many other domains of computer science as cryptography, formal verification, formal proof, artificial intelligence, *etc*.

We are convinced that this rapid expansion will undoubtedly change once again the everyday life of mathematicians within the next decades. For instance, it is quite possible that the verification of the correctness of the mathematical contents of papers will be soon left to computers (at least for the easiest parts). However many "pure" mathematicians are still ignoring these deep transformations and continue to work as they always did without taking advantage of the complete power of softwares and computers. One important objective of the CLap–CLap project is to participate in filling this gap.

Project impact

The CLap–CLap ANR project is a very ambitious research program of constructive and algorithmic approach of p-adic theory. The aim of the CLap–CLap ANR project is to bring together the necessary competence to achieve several ambitious targets. It gathers a large team of experts in several complementary fields from fundamental algebraic geometry to applied arithmetics. All the aspects of p-adic Hodge theory are represented: p-adic geometry, automorphic forms, Shimura varieties, integral lattices, representations theory, Galois representations...

The originality of this project is to put together experts of fundamental p-adic theory and implementation strategies. The goal is to pursue simultaneous progress and to compare the two approaches to inspire, guide new developments. The expected results are even all new fundamental theoretical knowledge and the diffusion of creative original ideas. Our intuition will be guided by explicit computations: using SageMath implementations, we are convinced we will be able to determine new deep phenomena in less than two years. The CLap-CLap project is a new step to put closer computer and researchers. Computer will be used to produce datas, objects of researchers in abstract mathematics too.

One other important aim of the CLap–CLap project is to generalize the use of algorithmical methods in number theory and especially in the Langlands program. We believe that we have to potential for initiating and accelerating this transformation. Indeed our team is composed of the present young (at most 50 years) and the next generations of professors and directors of research. Most of them are responsible for training Graduate students in a major French Institutions (University of Rennes, École Polytechnique, École Normale Supérieure de Lyon, University of Bordeaux, University of Paris Sud, University Pierre and Marie Curie...) and all are convinced and engaged to promote that computational approach is essential in the scientific training as weel as in the research in future. The impact force in France of our teams of young professors is very important: all together, we are able to reach more than 200 French students in Master of Mathematics. We will systematically expose them to a systematic use of computer as an everyday ally for a successful research activity.

The CLap–CLap project gathers the most famous french mathematicians working in the topics of p-adic Langlands correspondence and computer algebra. It then has the ambition to become an unavoidable actor in these domains. It will moreover contribute to the organization of the French research lattice. It will allow to develop the scientific cohesion of the three poles Lyon–Rennes–Paris during the two first years. During the third year, we will crystallize all the national p-adic strengh united around a commun project of trained trimester. The last year of the project will be magnified through this trimester (2022) and its aftershocks.

Strategy of valorisation

CLap–CLap ANR members will make a special effort to disseminate their results. Publications will be a priority goal of this project. Graduate students will be enrolled to the internal workgroups. These workgroups have vocation to crystallise all the potential local energies in the field. The members of the CLap–CLap project are renowned mathematicians who are often invited to international conferences. They will use these opportunities to report on the progress of the CLap–CLap project and try to convince collegues to rely more on computers for their research. As already discussed in §2.2, we will further organize two international conferences and one CLap–CLap trimester where the most influent number theorists will be invited. Talks and software demonstrations highlighting our works will be presented there.

We will maintain a website offering many attractive features and functionalities. We plan for instance to provide user-friendly solutions in order to allow everybody to test our softwares online without needing to install them. We will also maintain databases gathering all our "numerical" results and design a user-friendly web interface for requesting them.

Caruso is the bearer of an original project of short scientific videos introducing briefly an interesting topic in mathematics (see http://www.lebesgue.fr/5min). We will import there ideas into the framework of the CLap–CLap project: as a concrete example we plan to produce a short video (about 15 minutes) presenting each new contribution (papers and softwares) developed by the members of the CLap–CLap project.

Bibliographic references

[1] N. Abe, G. Henniart, F. Herzig, M.-F. Vignéras, *A classification of irreducible admissible mod p representations of p-adic reductive groups*, J. AMS (to appear)

[2] D. Abramovich, M. Romagny, *Moduli of Galois p-covers in mixed characteristics*, Algebra Number Theory **6** (2012), 757–780

[3] M. Agrawal, N. Kayal, N. Saxena, *PRIMES is in P.* Ann. of Math. **160** (2004), 781–793

[4] E. Berlekamp, *Factoring polynomials over finite fields*, Bell System Tech. J. **46** (1967), 1853–1859

[5] L. Berger, *Représentations modulaires de $\mathrm{GL}_2(\mathbb{Q}_p)$ et représentations galoisiennes de dimension 2*, Astérisque **330** (2010), 263–279

[6] L. Berger, C. Breuil, *Sur quelques représentations potentiellement cristallines de $\mathrm{GL}_2(\mathbb{Q}_p)$*, Astérisque **330** (2010), 155–211

[7] I. Bouw, J. Cooley, K. Lauter, E. Lorenzo García, M. Manes, R. Newton, E. Ozman, *Bad reduction of genus 3 curves with complex multiplication*, joint work with Women in Numbers Europe, research Directions in Number Theory, Springer (2015), 109–151

[8] J. Breeding, C. Poor, D. Yuen, *Computations of Spaces of Paramodular Forms of General Level*, J. of Korean Math. Soc. (to appear)

[9] C. Breuil, *Sur quelques représentations modulaires et p-adiques de $\mathrm{GL}_2(\mathbb{Q}_p)$ I*, Compositio Math. **138** (2003), 165–188

[10] C. Breuil, *Invariant $\mathcal{L}$ et série spéciale p-adique*, Ann. Scient. É. N. S. **37** (2004), 559–610

[11] C. Breuil, *Série spéciale p-adique et cohomologie étale complétée*, Astérisque **331** (2010), 65–115

[12] C. Breuil, *Vers le socle localement analytique II*, Math. Annalen **361** (2015), 741–785

[13] C. Breuil, Ext^1 *localement analytique et compatibilité local-global*, Amer. Journal of Math. (to appear)

[14] C. Breuil, F. Diamond, *Formes modulaires de Hilbert modulo p et valeurs d'extensions galoisiennes*, Ann. Scient. de l'E.N.S. **47** (2014), 905–974

[15] C. Breuil, E. Hellmann, B. Schraen, *Une interprétation modulaire de la variété trianguline*, Math. Annalen **367** (2017), 1587–1645.

[16] C. Breuil, E. Hellmann, B. Schraen, *Smoothness and classicality on eigenvarieties*, Inventiones Math. **209** (2017), 197–274.

[17] C. Breuil, E. Hellmann, B. Schraen, *A local model for the trianguline variety and applications*, preprint (2017)

[18] C. Breuil, F. Herzig, *Ordinary representations of $G(\mathbb{Q}_p)$ and fundamental algebraic representations*, Duke Math. J. **164** (2015), 1271–1352

[19] C. Breuil, A. Mézard, *Multiplicités modulaires et représentations de $\mathrm{GL}_2(\mathbb{Z}_p)$ et de $\mathrm{Gal}(\bar{\mathbb{Q}}_p/\mathbb{Q}_p)$ en $\ell = p$*, Duke Math. J. **115** (2002), 205–310

[20] C. Breuil, V. Paškūnas, *Towards a modulo p Langlands correspondence for* GL_2, Memoirs A.M.S. **1016** (2012)

[21] P. Bürgisser, M. Clausen, M. Amin Shokrollahi, *Algebraic complexity theory*, Grundlehren der Mathematischen Wissenschaften **315** (1997)

[22] A. Cadoret, A. Kret, *Galois generic points on Shimura varieties*, preprint (2016)

[23] A. Caraiani, M. Emerton, T. Gee, D. Geraghty, V. Paškūnas, S. W. Shin, *Patching and the p-adic local Langlands correspondence*, Compositio Math. (to appear)

[24] A. Caraiani, B. Levin, *Kisin modules with descent data and parahoric local models*, preprint (2015)

[25] X. Caruso, *Classification of integral models of* $(\mathbb{Z}/p^2\mathbb{Z})_K$ *via Breuil-Kisin theory*, J. of Algebra **323** (2010), 1955–1957

[26] X. Caruso, *p-adic precision*, librairie SAGEMATH (2013)

[27] X. Caruso, *Lattices in semi-stable representations*, librairie SAGEMATH (2013)

[28] X. Caruso, *Dimensions de certaines variétés de Kisin*, J. reine angew. Math. (to appear)

[29] X. Caruso, *Numerical stability of Euclidean algorithm over ultrametric fields*, J. Theor. Nomb. Bordeaux (to appear)

[30] X. Caruso, A. David, A. Mézard, *Un calcul d'anneaux de déformations potentiellement Barsotti–Tate*, Trans. AMS (to appear)

[31] X. Caruso, A. David, A. Mézard, *Variétés de Kisin stratifiées et déformations potentiellement Barsotti–Tate*, J. Inst. Math. Jussieu (2016), DOI 10.1017/S1474748016000232

[32] X. Caruso, D. Roe, T. Vaccon, *Tracking p-adic precision*, LMS J. Comput. Math. **17** (2014), 274–294

[33] X. Caruso, D. Roe, T. Vaccon, *p-adic stability in linear algebra*, proceedings of ISSAC'2015

[34] F. Cléry, C. Faber, G. van der Geer, *Covariants of binary sextics and vector-valued Siegel modular forms of genus two*, preprint (2016)

[35] H. Cohen, *Advanced topics in computational number theory*, Graduate Texts in Mathematics **193** (2000), Springer-Verlag, New York

[36] H. Cohen, F. Diaz y Diaz, M. Olivier, *A table of totally complex number fields of small discriminants*, Lecture Notes in Comput. Sci. **1423** (1998), 381–391

[37] H. Cohen, F. Diaz y Diaz, M. Olivier. *Tables of octic fields with a quartic subfield*, Math. Comp. **68** (1999), 1701–1716

[38] H. Cohen, H. Lenstra Jr, *Heuristics on class groups of number fields*, Lecture Notes in Math **1068** (1984), 33–62

[39] H. Cohen and al. *Handbook of elliptic and hyperelliptic curve cryptography*, Discrete Mathematics and its Applications (Boca Raton). Chapman & Hall/CRC, Boca Raton, FL, 2006.

[40] P. Colmez, *Représentations de* $\mathrm{GL}_2(\mathbb{Q}_p)$ *et* (φ,Γ)*-modules*, dans Représentations p-adiques de groupes p-adiques II, Astérisque **330** (2010), 281–509

[41] P. Colmez, *Correspondance de Langlands locale p-adique et changement de poids*, J. EMS (to appear)

[42] P. Colmez, G. Dospinescu, *Complétés universels de représentations de* $\mathrm{GL}_2(\mathbb{Q}_p)$, Algebra and Number Theory (to appear)

[43] P. Colmez, G. Dospinescu, W. Nizioł, *Cohomologie p-adique de la tour de Drinfeld: le cas de la dimension* 1, preprint (2017)

[44] P. Colmez, G. Dospinescu, W. Nizioł, *Cohomology of p-adic Stein spaces*, preprint (2017)

[45] P. Colmez, G. Dospinescu, V. Paškūnas, *The p-adic local Langlands correspondence for* $\mathrm{GL}_2(\mathbb{Q}_p)$, Cambridge J. Math. **2** (2014), 1–47.

[46] J. Conway, R. Curtis, S. Norton, R. Parker, R. Wilson, *Atlas of finite groups*, Oxford University Press (1985), Eynsham

[47] C. Cornut, M.-H. Nicole, *Cristaux et immeubles*, Bull. SMF (to appear)

[48] J.-M. Couveignes, R. Lercier, *The geometry of some parameterizations and encodings*, Adv. Math. Comm. **8** (2014), 437–458

[49] J. Cremona, *Algorithms for modular elliptic curves*, Cambridge University Press (1997), Cambridge, second edition

[50] J. Cremona, *Computing periods of cusp forms and modular elliptic curves*, Experiment. Math. **6** (1997), 97–107

[51] A. David, *Critère d'irréductibilité pour les courbes elliptiques semi-stables sur un corps de nombres*, preprint (2012)

[52] A. David, *Poids de Serre dans la conjecture de Breuil–Mézard*, preprint (2013)

[53] G. Dospinescu, A.-C. Le Bras, *Revêtements du demi-plan de Drinfeld et correspondance de Langlands p-adique*, Ann. of Math. **186** (2017), 321–411

[54] B.Edixhoven, J.-M. Couveignes, *Computational aspects of modular forms and Galois representations*, Annals of Mathematics Studies **176** (2001)

[55] M. Emerton, *A local-global compatibility conjecture in the p-adic Langlands programme for* $\mathrm{GL}_{2/\mathbb{Q}}$, Pure and Applied Math. Quarterly **2** (2006), 279–393

[56] M. Emerton, *Local-global compatibility in the p-adic Langlands programme for* $\mathrm{GL}_{2/\mathbb{Q}}$, preprint (2011)

[57] A. Enge, E. Thomé, *Computing class polynomials for abelian surfaces*, Experimental Mathematics, **23**, (2014), 129–145

[58] L. Fargues, *Quelques résultats et conjectures concernant la courbe*, Astérisque **369** (2015), 325–374

[59] T. Gee, F. Herzig, D. Savitt, *General Serre weight conjectures*, J. EMS (to appear)

[60] T. Gee, M. Kisin, *The Breuil–Mézard conjecture for potentially Barsotti-Tate representations*, Forum of Math, Pi **2** (2014), 56 pp.

[61] A. Ghitza, N. Ryan, D. Sulon, *Computations of vector-valued Siegel modular forms*, J. Number Theor. **133** (2013), 3921–3940

[62] A. Ghitza, A. McAndrew, *Theta operators on Siegel modular forms and Galois representations*, preprint (2016)

[63] E. Grosse-Klönne, *From pro-p Iwahori-Hecke modules to (φ, Γ)-modules I*, Duke Math. J. (to appear).

[64] M. Harris, R. Taylor, *The geometry and cohomology of some simple Shimura varieties*, Ann. Math. Study **151** (2001), Princeton University Press

[65] J. Hauseux, *Extensions entre séries principales p-adiques et modulo p de $G(F)$*, J. Inst. Math. Jussieu 15 (2016), no. 2, 225–270

[66] J. Hauseux, *Sur une conjecture de Breuil–Herzig*, J. Reine Angew. Math. (to appear)

[67] J. Hauseux, *Parabolic induction and extensions*, Algebra and Number Theory (to appear)

[68] J. Hauseux, T. Schmidt, C. Sorensen, *Deformation rings and parabolic induction*, J. Théor. Nombres Bordeaux (to appear)

[69] J. Hauseux, T. Schmidt, C. Sorensen, *Functorial properties of generalized Steinberg representations*, preprint (2017)

[70] E. Hellmann, *Families of trianguline representations and finite slope spaces*, preprint (2012)

[71] E. Hellmann, B. Schraen, *Density of potentially crystalline representations of fixed weight*, Compositio Math. (to appear)

[72] G. Henniart, *Une preuve simple des conjectures de Langlands pour GL_n sur un corps p-adique*, Invent. Math. **139** (2000), 439–455

[73] F. Herzig, *The classification of irreducible admissible mod. p representations of a p-adic GL_n*, Invent. Math. **186** (2011), 373–434

[74] F. Herzig, D. Le, S. Morra, *On mod p local-global compatibility for GL_3 in the ordinary case*, Compositio Math. **153** (2017), 2215–2286

[75] Y. Hu, S. Morra, B. Schraen, *Sur la fidélité de certaines représentations de $\mathrm{GL}_2(F)$ sous une algèbre d'Iwasawa*, Rend. Semin. Mat. Univ. Padova. **131** (2014), 49–65

[76] C. Huyghe, D. Patel, M. Strauch, T. Schmidt, *$D^\dagger$-affinity of formal models of flag varieties*, preprint (2015)

[77] M. Kisin, *Potentially semi-stable deformation rings*, J. Amer. Math. Soc. **21** (2008), 513–546

[78] M. Kisin, *The Fontaine-Mazur conjecture for GL_2*, J. Amer. Math. Soc. **22** (2009), 641–690

[79] M. Kisin, *Deformations of $\mathrm{Gal}(\bar{\mathbb{Q}}_p/\mathbb{Q}_p)$ and $\mathrm{GL}_2(\mathbb{Q}_p)$ representations*, dans Représentations p-adiques de groupes p-adiques II, Astérisque **330** (2010), 511–528

[80] J. K. Koo, D. H. Shin, D. S. Yoon, *Generators of Siegel modular function field of higher genus and level*, preprint (2016)

[81] D. Le, B. Le Hung, B. Levin, S. Morra, *Potentially crystalline deformation rings: shapes and shadows*, Invent. Math. (to appear)

[82] F. Lemma, T. Ochiai, *The Coleman map for Hida families of GSp(4)*, Amer. Journ. of Math **136** (2014), 729–760

[83] F. Lemma, *On higher regulators of Siegel threefolds I: the vanishing on the boundary*, Asian Journ. of Math **19** (2015), 83–120

[84] R. Lercier, C. Ritzenthaler, J. Sijsling, *Fast computation of isomorphisms of hyperelliptic curves and explicit descent*, Proceedings of the Tenth Algorithmic Number Theory Symposium (2013), 463–486

[85] R. Lercier, C. Ritzenthaler, F. Rovetta, J. Sijsling, *Parametrizing the moduli space of curves and applications to smooth plane quartics over finite fields*, LMS J. Comp. Math. (to appear)

[86] R. Lercier, C. Ritzenthaler, J. Sijsling, *Explicit Galois obstruction and descent for hyperelliptic curves with tamely cyclic reduced automorphism group* Math. Comp. (to appear)

[87] R. Lercier, C. Ritzenthaler, *Hyperelliptic curves and their invariants: Geometric, arithmetic and algorithmic aspects*, J. Algebra **372** (2012), 595–636

[88] A. Logan, D. McKinnon, R. van Luijk, *Density of rational points on diagonal quartic surfaces*, Algebra Number Theory **4** (2010), 1–20

[89] D. Lubicz, D. Robert, *Computing isogenies between abelian varieties*, Compositio Math. **148** (2012), 1483–1515

[90] D. Madore, F. Orgogozo, *Calculabilité de la cohomologie étale modulo ℓ*, Algebra Number Theory **9** (2015), 1647–1739

[91] J. Martinet, *Modules sur l'algèbre du groupe quaternionien*, Ann. Sci. École Norm. Sup. **4** (1971), 399–408

[92] J. Martinet, *Perfect lattices in Euclidean spaces*, Grundlehren der Mathematischen Wissenschaften **327** (2003), Springer-Verlag, Berlin

[93] N. Mascot, *Computing modular Galois representations*, PhD thesis, available at http://www.theses.fr/2014BORD0108

[94] N. Mascot, *Computing modular Galois representations*, Rendiconti del Circolo Matematico di Palermo **62** (2013), 451–476

[95] N. Mascot, *Certifying modular Galois representations*, preprint (2015)

[96] A. Mézard, M. Romagny, D. Tossici, *Models of group schemes of roots of unity*, Ann. Inst. Fourier (Grenoble) 63 (2013), no. 3, 1055–1135

[97] M. Monge, *A constructive theory for extensions of p-adic fields*, PhD thesis (2012)

[98] S. Mossige, *Table of irreducible polynomials over* $\mathrm{GF}[2]$ *of degrees* 10 *through* 20, Math. Comp. **26** (1972), 1007–1009

[99] E. Nart, C. Ritzenthaler, *A new proof of Thomae-like formula for non hyperelliptic genus three curves*, proceedings of AGCT'14

[100] A. Odlyzko, *The* 10^{22}*-nd zero of the Riemann zeta function*, Contemp. Math. **290** (2001), 139–144

[101] A. Page, *Méthodes explicites pour les groupes arithmétiques*, PhD Thesis (2014)

[102] C. Papadimitriou, *Computational complexity*, Addison-Wesley Publishing Company (1994), Reading, MA

[103] C. Park, Z. Qian, *On mod p local-global compatibility for* $\mathrm{GL}_n(\mathbb{Q}_p)$ *in the ordinary case*, preprint (2017)

[104] V. Paškūnas, *Coefficient systems and supersingular representations of* $\mathrm{GL}_2(F)$, Mémoires S.M.F. **99** (2004)

[105] V. Paškūnas, *The image of Colmez's Montréal functor*, Pub. Math. I.H.É.S. **118** (2013), 1–191

[106] D. Patel, T. Schmidt, M. Strauch *Locally analytic representations of* $\mathrm{GL}_2(L)$ *via semistable models of* $\mathbb{P}^1$, J. Inst. Math. Jussieu, DOI 10.1017/S1474748016000396

[107] C. Poor, D. Yuen, *Paramodular Cusp Forms*, Math. Comp. **84** (2015), 1401–1438.

[108] M. Romagny, *Effective models of group schemes*, J. Algebraic Geom. **21** (2012), 643–682

[109] The Sage Developers, *Sage Mathematics Software (Version x.y.z)*, YYYY, `http://www.sagemath.org`

[110] T. Schmidt, *On unitary deformations of smooth modular representations*, Israel J. Math. **193** (2013), 15–46

[111] T. Schmidt, *Hecke algebras and affine flag varieties in characteristic p*, J. Pure Appl. Algebra **220** (2016), 3233–3247

[112] P. Schneider, J. Teitelbaum, *Algebras of p-adic distributions and admissible representations*, Invent. Math. **153** (2003), 145–196

[113] P. Scholze, *On the p-adic cohomology of the Lubin-Tate tower*, Annales ENS (to appear)

[114] B. Schraen, *Représentations localement analytiques de* $\mathrm{GL}_3(\mathbb{Q}_p)$ *et invariants* $\mathcal{L}$, Ann. Scient. É. N. S. **44** (2011), 43–145

[115] B. Schraen, *Sur la présentation des représentations supersingulières de* $\mathrm{GL}_2(F)$, J. Reine Angew. Math. **704** (2015), 187–208

[116] D. Shanks, *The simplest cubic fields*, Math. Comp. **28** (1974), 1137–1152

[117] C. Sorensen, *Deformations of principal series modulo p for* GL_n, preprint (2015)

[118] W. Stein, *Studying the Birch and Swinnerton-Dyer conjecture for modular abelian varieties using Magma*, Algorithms Comput. Math. **19** (2006), 93–116

[119] O. Taibi, *Dimensions of spaces of level one automorphic forms for split classical groups using the trace formula*, Ann. Scient. ENS (to appear)

[120] T. Vaccon, *Matrix-F5 algorithms over finite-precision complete discrete valuation fields*, J. Symb. Comp **80** (2017), 329–350

[121] M.-F. Vignéras, *ℓ-adic Banach representations of reductive p-adic groups*, dans Représentations p-adiques de groupes p-adiques II, Astérisque **330** (2010), 1–11

[122] J. Voight, *Computing automorphic forms on Shimura curves over fields with arbitrary class number*, Lecture Notes in Comput. Sci. **6197** (2010), 357–371

[123] J. Voight, J. Willis, *Computing power series expansions of modular forms*, Contrib. Math. Comput. Sci. **6** (2014), 331–361

[124] J. von zur Gathen, J. Gerhard. *Modern computer algebra*, Cambridge University Press (2013), Cambridge, third edition

[125] L. Walling, *A formula for the action of Hecke operators on half-integral weight Siegel modular forms and applications*, J. Number Theor. **133** (2013), 1608–1644

[126] L. Walling, *Explicit matrices for Hecke operators on Siegel modular forms*, preprint (2007)

[127] H. Williams, C. Zarnke, *A report on prime numbers of the forms* $M = (6a+1)2^{2m-1} - 1$ *and* $M' = (6a-1)2^{2m} - 1$, Math. Comp. **22** (1968), 420–422